

STRUCTURED DATA PROCESSING QUALITY CONTROL AND QUANTITY MODELLING OF PRODUCTS ON A PRODUCTION LINE

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ABSTRACT

The destination of this paper is structured data processing quality control and quantity modelling of products on a production line & to analyze how production system design, quality, and productivity of product are inter-related in small production systems. We develop a new process model for machines with both quality and operational failures, and we identify important deference's between types of quality failures. We also develop models for two-machine systems, with in nite buers, buers of size zero, and -nite buers. We calculate total production rate, effective production rate (i e, the production rate of good parts), and yield. Numerical studies using these models show that when the first machine has quality failures and the inspection occurs only at the second machine, there are cases in which the effective production rate increases as buer sizes increase, and there are cases in which the effective production rate decreases for larger buers. We propose extensions to larger systems. The success of the Car Production System has prodding much research in manufacturing systems engineering. Productivity and quality have been extensively studied, but there is little research in their overlap.

Key words: Data Processing Quality, Productivity and Manufacturing System Design, buffer Design, Production, and Quantity of product.

INTRODUCTION

The recent popularity of Statistical Quality Control (SQC), Total Quality Management (TQM), and Six Sigma have demonstrated the importance of quality.

These two ends, productivity and quality, have been extensively studied and reported separately both in the manufacturing systems research literature and the practitioner literature, but there is little research in their intersection. The need for such work was recently described by authors from the GM Corporation based on their experience [13]. All manufacturers must satisfy these two requirements (high productivity and high quality) at the same time to maintain their competitiveness.

Toyota Production System advocates admonish factory designers to combine inspections with operations. In the T oyota Production System, the machines are designed to detect abnormalities and to stop automatically whenever they occur. Also, operators are equipped with means of stopping the production flow whenever they note anything suspicious. (They call this practice

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jidoka.) Toyota Production System advocates argue that mechanical and human jidokaa prevent the waste that would result from producing a series of defective items. Therefore jidokaa is a means to improve quality and increase productivity at the same time [23], [24]. But this statement is arguable: quality failures are often those in which the quality of each part is independent of the others. This is the case when the defect takes place due to common (or chance or random) causes of variations [16]. In this case, there is no reason to stop a machine that has made a bad part because there is no reason to believe that stopping it will reduce the number of bad parts in the future. In this case, therefore, stopping the operation does Sequence quality but it does reduce productivity. On the other hand, when quality failures are those in which once a bad part is produced, all

subsequent parts will be bad until the machine is repaired (due to special or assignable or systematic causes of Variations) [16], catching bad parts and stopping the machine as soon as possible is the best way to maintain high quality and productivity. Non-stock or lean production is another popular buzzword in manufacturing systems engineering. Some lean manufacturing professionals advocate reducing inventory on the factory floor since the reduction of work-in-process (WIP) reveals the problems in the production lines [3]. In fact, Toyota recently changed their view on inventory and are trying to re-adjust their inventory levels [9]. What is missing in discussions of factory design, quality, and productivity is a quantitative model to show how they are inter-related. Most of the arguments about this are based on anecdotal evidence or qualitative reasoning that lacks a sound scientific quantitative foundation. The research described here tries to establish such a foundation to investigate how production system design and operation in Productivity and product quality by developing conceptual and computational models of two-machine-one-buffer systems and performing numerical experiments.

HISTORY

This is inherent in the design of the process. Such failures can be represented by independent Bernoulli random variables, in which a binary random variable, which indicates whether or not the part is good, is chosen each time a part is operated on. A good part is produced with probability, and a bad part is produced with probability 1. The occurrence of a bad part implies nothing about the quality of future parts, so no permanent changes can have occurred in the machine's or the sake of clarity, we call this a Bernoulli-type quality failure. Most of the quantitative literature on inspection allocation assumes this kind of quality failure [21]. In this case, if bad parts are destined to be scrapped, it is useful to catch them as soon as possible because the longer before they are scrapped; the more they consume the capacity of downstream machines. However, there is no reason to stop a machine that has produced a bad part due to this kind of failure.

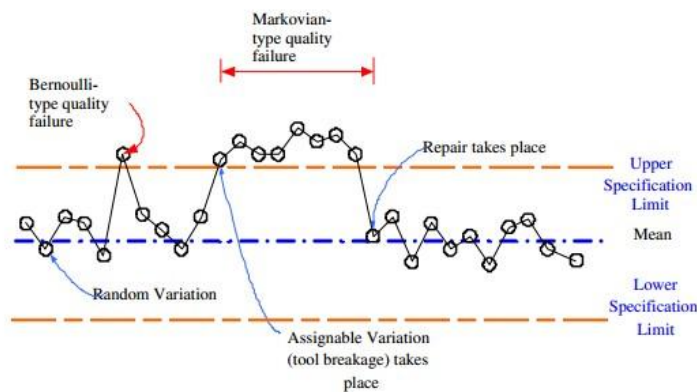


Fig.1 Types of Failures

System yield is defined here as the fraction of input to a system that is transformed into output of acceptable quality. This is an important metric because customers observe the quality of products only after all the manufacturing processes are done and the products are shipped the system yield is a complex function of how the factory is designed and operated, as well as of the characteristics of the machines. Some of factors include individual operation yields, inspection strategies, operation policies, buer sizes, and other factors.

OUTLINE

In Section 2 we introduce the structure of the modelling techniques used in this paper. We present modelling, solution techniques, and validation of the 2-machine-1- nitebuer case in Section 3. Discussions on the behaviour of a production line based on numerical experiments are provided in Section 5. A future research plan is shown in Section 6. Parameters of many of the systems studied numerically here, and details of the analytical solution of the two-machine line, can be found in the appendices.

MATHEMATICAL MODELS

Single machine model there are many possible ways to characterize a machine for the purpose of simultaneously studying quality and quantity issues. Here, we model a machine as a discrete state, continuous time Markov process. Material is assumed continuous, and speed at which Machine processes material while it is operating and not constrained by the other machine or the buer. It is a constant, in that μ_i does not depend on the repair state of the other machine or the buer level.

Figure 2 shows the proposed state transitions of a single machine with persistent-type quality failures. In the model, the machine has three states:

State 1: The machine is operating and producing good parts.

State -1: The machine is operating and producing bad parts, but the operator does not know this yet.

State 0: The machine is not operating.

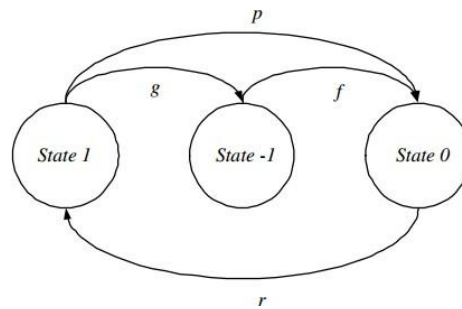


Fig.2 states of a machine

The machine therefore has two deferent failure modes (i.e. transition to failure states from state 1):

- Operational failure: transition from state 1 to state 0. The machine stops producing parts due to failures like motorburnout.
- Quality failure: transition from state 1 to state -1. The machine stops producing good parts (and starts producing bad parts) due to a failure like a sudden tooldamage.

When a machine is in state 1, it can fail due to a non-quality related event. It goes to state 0 with transition probability rate p . After that an operator fixes it, and the machine goes back to state 1 with transition rate r . Sometimes, due to an assignable cause, the machine begins to produce bad parts, so there is a transition from state 1 to state -1 with a probability rate g . Here g is the reciprocal of the Mean Time to Quality Failure (MTQF). A more stable operation leads to a larger MTQF and a smaller g . The machine, when it is in state -1, can be stopped for two reasons: it may experience the same kind of operational failure as it does when it is in state 1; and the operator may stop it for repair when he learns that it is producing bad parts. The transition from state -1 to state 0 occurs at probability rate $f = p + h$ where h is the reciprocal of the Mean Time To Detect (MTTD). A more reliable inspection leads to a shorter MTTD and a larger f . (The detection can take place elsewhere, for example at a remote inspection station.) Note that this implies that $f > p$. Here, for simplicity, we assume that whenever a machine is repaired, it goes back to state 1. All the indicated transitions are assumed to follow exponential distributions. Single Machine Analysis To determine the production rate of a single machine, we first determine the steady-state probability distribution. This is calculated based on the probability balance principle: the probability of leaving a state is the same as the probability of entering that state. We have

$$(g + p)P(1) = rP(0) \quad (1)$$

$$fP(-1) = gP(1) \quad (2)$$

$$rP(0) = pP(1) + fP(-1) \quad (3)$$

The probabilities must also satisfy the normalization equation:

$$P(0) + P(1) + P(-1) = 1 \quad (4)$$

The solution of (1)–(4) is

$$P(1) = \frac{1}{1 + (p + g)/r + g/f} \quad (5)$$

$$P(0) = \frac{(p + g)/r}{1 + (p + g)/r + g/f} \quad (6)$$

$$P(-1) = \frac{g/f}{1 + (p + g)/r + g/f} \quad (7)$$

The total production rate, including good and bad parts, is

$$P_T = \mu(P(1) + P(-1)) = \mu \frac{1 + g/f}{1 + (p + g)/r + g/f} \quad (8)$$

The effective production rate, the production rate of good parts only, is

$$P_E = \mu P(1) = \mu \frac{1}{1 + (p + g)/r + g/f} \quad (9)$$

The yield is

$$\frac{P_E}{P_E + P_T} = \frac{P(1)}{P(1) + P(-1)} = \frac{f}{f + g} \quad (10)$$

A flow (or transfer) line is a manufacturing system with a very special structure. It is a linear network of service stations or machines (M_1, M_2, \dots, M_k) separated by buffer storages (B_1, B_2, \dots, B_{k-1}). Material flows from outside the system to M_1 , then to B_1 , then to M_2 , and so forth until it reaches M_k , after which it leaves. Figure 3 depicts a flow line. The rectangles represent machines and the circles represent buffers.

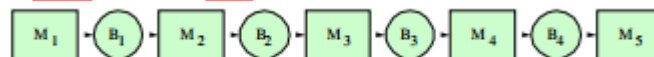


Fig.3 five machine flow line

2-machine-1-buffer (2M1B) models should be studied first. Then a de-composition technique, that divides a long transfer line into multiple 2-machine-1-buffer models, could be developed. (See [14].) Among the Various modelling techniques for the 2M1B case, including deterministic, exponential, and continuous models, the continuous material line model is used for this research because it can handle deterministic but different operation times at each operation. This is an extension of the continuous material serial line modelling of [10] by adding another machine failure state. Figure 4 shows the 2M1B continuous model where the machines, buffer and discrete parts are represented as valves, a tank, and a continuous fluid.

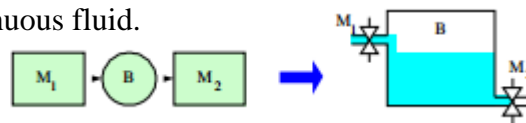


Fig.4 Two-Machine-One-Buffer Continuous Model

INFINITE BUFFER CASE AN IN FINITE BUFFER CASE IS A SPECIAL 2M1B LINE IN WHICH THE SIZE OF THE BUFFER

(B) is infinite. This is an extreme case in which the first machine (M1) never suffers from blockage. To derive expressions for the total production rate and the effective production rate, we observe that when there is infinite buffer capacity between two machines (M1, M2), the total production rate of the 2M1B system is a minimum of the total production rates of M1 and M2. The total production rate of machine i is given by (8), so the total production rate of the 2M1B system is the feature that M1 works on could take place at an inspection station at M2, and this inspection could trigger a repair of M1. (We call this quality

information feedback. See Section 4.) In that case, the MTTD of M1 (and therefore f_1) will be a function of the amount of material in the buffer. We return to this important case in Section 4

Table 1 Validation of Infinite Buffer Case

Case #	P_T^{∞} (Analytic)	P_T^{∞} (Simulation)	%Difference
1	0.762	0.761	0.17
2	0.708	0.708	0.00
3	0.657	0.657	0.00
4	0.577	0.580	-0.50
5	0.527	0.530	-0.42
6	0.745	0.745	0.01
7	0.762	0.760	0.30
8	1.524	1.522	0.14
9	0.762	0.762	0.00
10	1.524	1.526	-0.13

$$P_T^{\infty} = \min \left[\frac{\mu_1(1 + g_1/f_1)}{1 + (p_1 + g_1)/r_1 + g_1/f_1}, \frac{\mu_2(1 + g_2/f_2)}{1 + (p_2 + g_2)/r_2 + g_2/f_2} \right] \quad (11)$$

The probability that machine M_i does not add non-conformities is

$$Y_i = \frac{P_i(1)}{P_i(1) + P_i(-1)} = \frac{f_i}{f_i + g_i} \quad (12)$$

Since there is no scrap and rework in the system, the system yield becomes

$$\frac{f_1 f_2}{(f_1 + g_1)(f_2 + g_2)} \quad (13)$$

As a result, the effective production rate is

$$P_E^{\infty} = \frac{f_1 f_2}{(f_1 + g_1)(f_2 + g_2)} P_T^{\infty} \quad (14)$$

The effective production rate evaluated from (14) has been compared with a discrete-event, discrete-part simulation. Table 1 shows good agreement. The parameters for these cases are shown in Appendix B.

ZERO BUFFER CASE

The zero buffer case is one in which there is no buffer space between the machines. This is the other extreme case where blockage and starvation take place most frequently. In the zero-buffer case in which machines have different operation times, whenever one of the machines stops, the other one is also stopped. In addition, when both of them are working, the production rate is $\min[\mu_1; \mu_2]$. Consider a long time interval of length T during which M1 fails m_1 times and M2 fails m_2 times. If we assume that the average time to repair M1 is $\mu_1 = r_1^{-1}$ and the average time to repair M2 is $\mu_2 = r_2^{-1}$, then the total system downtime will be close to $D = m_1/r_1 + m_2/r_2$. Consequently, the total up time will be approximately

$$U = T - D = T - (m_1/r_1 + m_2/r_2) \quad (15)$$

Since we assume operation-dependent failures, the rates of failure are reduced for the faster machine. Therefore,

$$p_i^b = p_i \frac{\min(\mu_1, \mu_2)}{\mu_i}, \quad g_i^b = g_i \frac{\min(\mu_1, \mu_2)}{\mu_i}, \quad \text{and} \quad f_i^b = f_i \frac{\min(\mu_1, \mu_2)}{\mu_i} \quad (16)$$

The reduction of π is explained in detail in [10]. The reductions of g_i and f_i are done for the same reasons. Table 2 lists the possible working states 1 and 2 of M1 and M2. The third column is the probability of finding the system in the indicated state. The fourth and fifth columns indicate the expected number of transitions to down states during the time interval from each of the states in column 1.

Table 2 Zero-Buffer States, Probabilities, and Expected Numbers of Events

α_1	α_2	Probability $\pi(\alpha_1, \alpha_2)$	$Em_1(\alpha_1, \alpha_2)$	$Em_2(\alpha_1, \alpha_2)$
1	1	$\frac{f_1^b}{f_1^b + g_1^b} \frac{f_2^b}{f_2^b + g_2^b}$	$p_1^b U \pi(1, 1)$	$p_2^b U \pi(1, 1)$
1	-1	$\frac{f_1^b}{f_1^b + g_1^b} \frac{g_2^b}{f_2^b + g_2^b}$	$p_1^b U \pi(1, -1)$	$f_2^b U \pi(1, -1)$
-1	1	$\frac{g_1^b}{f_1^b + g_1^b} \frac{f_2^b}{f_2^b + g_2^b}$	$f_1^b U \pi(-1, 1)$	$p_2^b U \pi(-1, 1)$
-1	-1	$\frac{g_1^b}{f_1^b + g_1^b} \frac{g_2^b}{f_2^b + g_2^b}$	$f_1^b U \pi(-1, -1)$	$f_2^b U \pi(-1, -1)$

From Table 2, the expectations of m_1 and m_2 are

$$\begin{aligned} Em_1 &= \sum_{\alpha_1=-1}^1 \sum_{\alpha_2=-1}^1 Em_1(\alpha_1, \alpha_2) = \frac{U f_1^b (p_1^b + g_1^b)}{f_1^b + g_1^b} \\ Em_2 &= \sum_{\alpha_1=-1}^1 \sum_{\alpha_2=-1}^1 Em_2(\alpha_1, \alpha_2) = \frac{U f_2^b (p_2^b + g_2^b)}{f_2^b + g_2^b} \end{aligned} \quad (17)$$

By plugging them into equation (15), we find total production rate:

$$P_T^0 = \frac{\min[\mu_1, \mu_2]}{1 + \frac{f_1^b (p_1^b + g_1^b)}{r_1 (f_1^b + g_1^b)} + \frac{f_2^b (p_2^b + g_2^b)}{r_2 (f_2^b + g_2^b)}} \quad (18)$$

The effective production rate is

$$P_E^0 = \frac{f_1^b f_2^b}{(f_1^b + g_1^b)(f_2^b + g_2^b)} P_T^0 \quad (19)$$

The comparison with simulation is shown in in Table 3. The parameters of the cases are shown in Appendix B.

Table 3 Zero Buffer Case

Case #	$P_E^0(\text{Analytic})$	$P_E^0(\text{Simulation})$	%Difference
1	0.657	0.662	-0.73
2	0.620	0.627	-1.15
3	0.614	0.621	-1.03
4	0.529	0.534	-0.99
5	0.480	0.484	-0.77
6	0.647	0.651	-0.57
7	0.706	0.712	-0.91
8	1.377	1.406	-2.10
9	0.706	0.711	-0.77
10	1.377	1.380	-0.22

-Machine-1-Finite-Buffer Line The two-machine line is the simplest non-trivial case of a production line. In the existing literature on the performance evaluation of systems in which quality is not considered, two-machine lines are used in decomposition approximations of longer lines. (See [10].) We define the model here and show the solution technique in Appendix A

DEFINITION

The state of the 2M1B line is defined as $(x; 1; 2)$ where

- * x : the total amount of material in buffer B , $0 \leq x \leq N$,
- * α_1 : the state of M_1 . ($\alpha_1 = -1, 0$, or 1),
- * α_2 : the state of M_2 . ($\alpha_2 = -1, 0$, or 1)

The parameters of machine M_i are $\mu_i, r_i, p_i, f_i, g_i$ and the buffer size is N .

MODEL DEVELOPMENT

Internal transition equations

When buffer B is neither empty nor full, its level can rise or fall depending on the states of adjacent machines. Since it can change only a small amount during a short time interval, it is natural to use differential equations to describe its behaviour. The probability of finding both machines at state 1 with a total storage level between x and $x + \delta x$ at time $t + \delta t$ is given by $f(x; 1; 1) \delta t$, where

$$f(x, 1, 1, t + \delta t) = \{1 - (p_1 + g_1 + p_2 + g_2)\delta t\}f(x + (\mu_2 - \mu_1)\delta t, 1, 1) + r_2\delta t f(x - \mu_1\delta t, 1, 0) + r_1\delta t f(x + \mu_2\delta t, 0, 1) + o(\delta t) \quad (20)$$

This is because if both machines are at state 1 at time t and the storage level is between x and $x + (2 - \mu_1)\delta t$, then there should be no failures before $t + \delta t$ to get $f(x; 1; 1) \delta t$. The probability of not having any failures between t and $t + \delta t$ is

$$\{1 - (p_1 + g_1)\delta t\}\{1 - (p_2 + g_2)\delta t\} \simeq \{1 - (p_1 + g_1 + p_2 + g_2)\delta t\} \quad (21)$$

Probabilities for transitions from the states $(x - \mu_1\delta t; 1; 0)$ and $(x + \mu_2\delta t; 0; 1)$ to $(x; 1; 1)$ can be found similarly. After linearizing and letting $\delta t \rightarrow 0$, this equation becomes

$$\frac{\partial f(x, 1, 1)}{\partial t} = (\mu_2 - \mu_1) \frac{\partial f(x, 1, 1)}{\partial x} - (p_1 + g_1 + p_2 + g_2)f(x, 1, 1) + r_2f(x, 1, 0) + r_1f(x, 0, 1) \quad (22)$$

In steady state $\frac{\partial f}{\partial t} = 0$. Then, we have

$$(\mu_2 - \mu_1) \frac{df(x, 1, 1)}{dx} - (p_1 + g_1 + p_2 + g_2)f(x, 1, 1) + r_2f(x, 1, 0) + r_1f(x, 0, 1) = 0 \quad (23)$$

In the same way, the eight other internal transition equations for the probability density function are

$$p_2f(x, 1, 1) - \mu_1 \frac{df(x, 1, 0)}{dx} - (p_1 + g_1 + r_2)f(x, 1, 0) + f_2f(x, 1, -1) + r_1f(x, 0, 0) = 0 \quad (24)$$

$$g_2 f(x, 1, 1) + (\mu_2 - \mu_1) \frac{df(x, 1, -1)}{dx} - (p_1 + g + f_2) f(x, 1, -1) + r_1 f(x, 0, -1) = 0 \quad (25)$$

$$p_1 f(x, 1, 1) + \mu_2 \frac{df(x, 0, 1)}{dx} - (r_1 + p_2 + g_2) f(x, 0, 1) + r_2 f(x, 0, 0) + f_1 f(x, -1, 1) = 0 \quad (26)$$

$$p_1 f(x, 1, 0) + p_2 f(x, 0, 1) - (r_1 + r_2) f(x, 0, 0) + f_2 f(x, 0, -1) + f_1 f(x, -1, 0) = 0 \quad (27)$$

$$p_1 f(x, 1, -1) + g_2 f(x, 0, 1) - (r_1 + f_2) f(x, 0, -1) + \mu_2 \frac{df(x, 0, -1)}{dx} + f_1 f(x, -1, -1) = 0 \quad (28)$$

$$g_1 f(x, 1, 1) - (p_2 + g_2 + f_1) f(x, -1, 1) + (\mu_2 - \mu_1) \frac{df(x, -1, 1)}{dx} + r_2 f(x, -1, 0) = 0 \quad (29)$$

$$g_1 f(x, 1, 0) - \mu_1 \frac{df(x, -1, 0)}{dx} - (r_2 + f_1) f(x, -1, 0) + p_2 f(x, -1, 1) + f_2 f(x, -1, -1) = 0 \quad (30)$$

$$P(0, 1, 1, t + \delta t) = \{1 - (p_1 + g_1 + p_2^b + g_2^b) \delta t\} P(0, 1, 1) + r_1 P(0, 0, 1) \quad (32)$$

Boundary transition equations while the internal behaviour of the system can be described by probability density functions, there is a nonzero probability of finding the system in certain boundary states. For example, if $1 < 2$ and both machines are in state 1, the level of storage tends to decrease. If both machines remain operational for enough time, the storage will become empty ($x = 0$). Once the system reaches state $(0; 1; 1)$, it will remain there until a machine fails. There are 18 probability masses for boundary states $(P(N; 1; 2))$ and $P(0; 1; 2)$ where $1 = 1, 0$ or 1 , and $2 = 1, 0$ or 1 and 22 boundary equations for the $1 = 2$ case. To arrive at state $(0; 1; 1)$ at time $t + t$ when $1 = 2$, the system may have been in one of two states at time t . It could have been in state $(0; 1; 1)$ without any of operational failures and quality failures for both of machines. It could have been in state $(0; 0; 1)$ with a repair of the first machine. (The second machine could not have failed since it was starved). If the second order terms are ignored,

After the usual analysis, (32) becomes

$$\frac{\partial P(0, 1, 1)}{\partial t} = (p_1 + g_1 + p_2^b + g_2^b) P(0, 1, 1) + r_1 P(0, 0, 1) \quad (33)$$

In steady state

$$-(p_1 + g_1 + p_2^b + g_2^b) P(0, 1, 1) + r_1 P(0, 0, 1) = 0 \quad (34)$$

There are 21 other boundary equations derived similarly for $\mu_1 = \mu_2$ [14]:

$$P(0, 1, 0) = 0 \quad (35)$$

$$g_2^b P(0, 1, 1) - (p_1 + g_1 + f_2^b) P(0, 1, -1) + r_1 P(0, 0, -1) = 0 \quad (36)$$

$$p_1 P(0, 1, 1) - r_1 P(0, 0, 1) + \mu_2 f(0, 0, 1) + f_1 P(0, -1, 1) + r_2 P(0, 0, 0) = 0 \quad (37)$$

$$-(r_1 + r_2) P(0, 0, 0) = 0 \quad (38)$$

$$p_1 P(0, 1, -1) - r_1 P(0, 0, -1) + \mu_2 f(0, 0, -1) + f_1 P(0, -1, -1) = 0 \quad (39)$$

$$g_1 P(0, 1, 1) - (f_1 + p_2 + g_2) P(0, -1, 1) = 0 \quad (40)$$

$$-r_2P(N, -1, 0) + \mu_1f(N, -1, 0) + f_2P(N, -1, -1) + p_2P(N, -1, 1) = 0 \quad (50)$$

$$g_1^bP(N, 1, -1) + g_2P(N, -1, 1) - (f_1^b + f_2)P(N, -1, -1) = 0 \quad (51)$$

$$\mu_1f(0, 1, 0) = r_1P(0, 0, 0) + p_2^bP(0, 1, 1) + f_2^bP(0, 1, -1) \quad (52)$$

$$\mu_1f(0, -1, 0) = p_2^bP(0, -1, 1) + f_2^bP(0, -1, -1) \quad (53)$$

$$\mu_2f(N, 0, 1) = r_2P(N, 0, 0) + p_1^bP(N, 1, 1) + f_1^bP(N, -1, 1) \quad (54)$$

$$\mu_2f(N, 0, -1) = p_1^bP(N, 1, -1) + g_2P(N, 0, 1) + f_1^bP(N, -1, -1) \quad (55)$$

3.2.3 Normalization In addition to these, all the probability density functions and probability masses must satisfy the normalization equation:

$$\sum_{\alpha_1=-1,0,1} \sum_{\alpha_2=-1,0,1} \left[\int_0^N f(x, \alpha_1, \alpha_2)dx + P(0, \alpha_1, \alpha_2) + P(N, \alpha_1, \alpha_2) \right] = 1 \quad (56)$$

3.2.4 Performance measures After finding all probability density functions and probability masses, we can calculate the average inventory in the buffer from

$$\bar{x} = \sum_{\alpha_1=-1,0,1} \sum_{\alpha_2=-1,0,1} \left[\int_0^N xf(x, \alpha_1, \alpha_2)dx + NP(N, \alpha_1, \alpha_2) \right] \quad (57)$$

The total production rate is

$$P_T = P_T^1 = \sum_{\alpha_2=-1,0,1} \mu_1 \left[\int_0^N \{f(x, -1, \alpha_2) + f(x, 1, \alpha_2)\}dx + P(0, 1, \alpha_2) + P(0, -1, \alpha_2) \right] + \mu_2 \{P(N, 1, -1) + P(N, 1, 1) + P(N, -1, -1) + P(N, -1, 1)\} \quad (58)$$

The rate at which machine M_1 produces good parts is

$$P_E^1 = \sum_{\alpha_2=-1,0,1} \mu_1 \left[\int_0^N f(x, 1, \alpha_2)dx + P(0, 1, \alpha_2) \right] + \mu_2 \{P(N, 1, -1) + P(N, 1, 1)\} \quad (59)$$

The probability that the first machine produces a non-defective part is then $Y_1 = P_1^1 E = P_T$. Similarly, the probability that the second machine finishes its operation without adding a bad feature to a part is $Y_2 = P_2^1 E = P_T$,

Where

$$P_E^2 = \sum_{\alpha_1=-1,0,1} \mu_2 \left[\int_0^N f(x, \alpha_1, 1)dx + P(N, \alpha_1, 1) \right] + \mu_1 \{P(0, -1, 1) + P(0, 1, 1)\} \quad (60)$$

Therefore, the effective production rate is

$$P_E = Y_1 Y_2 P_T \quad (61)$$

VALIDATION

The 2M1B systems with the same machine speed ($1 = 2$) are solved in Appendix A. As we have indicated, we represent discrete parts in this model as a continuous fluid and time as a continuous variable. We compare analytical and simulation results in this section. In the simulation, both material and time are discrete. Details are presented in [14].

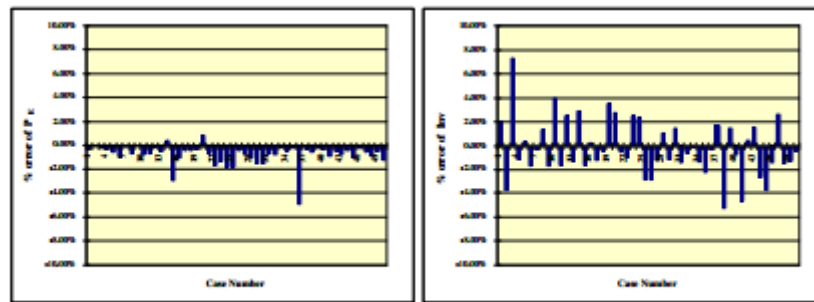


Fig. 5 Validation of the intermediate buffer size case

Figure 5 shows the comparison of the effective production rate and the average inventory from the analytic model and the simulation. 50 cases are generated by changing machines and buffer parameters and % errors are plotted in the vertical axis. The parameters for these cases are given in Appendix B. The % error in the effective production rate is calculated from

$$P_E \% \text{error} = \frac{P_E(A) - P_E(S)}{P_E(S)} \times 100(\%) \quad (62)$$

where $PE(A)$ and $PE(S)$ are the effective production rates estimated from the analytical model and the simulation respectively. But the % error in the average inventory is calculated from

$$Inv_E \% \text{error} = \frac{Inv_E(A) - Inv_E(S)}{0.5 \times N} \times 100(\%) \quad (63)$$

where $Inv_E(A)$ and $Inv_E(S)$ are average inventory estimated from the analytical model and the simulation respectively and N is a buffer size. The average absolute value of the % error in the effective production rate estimation is 0.76% and it is 1.89% for average inventory estimation.

QUALITY INFORMATION FEEDBACKS

Factory designers and managers know that it is ideal to have inspection after every operation. However, it is often costly to do this. As a result, factories are usually designed so that multiple inspections are performed at a small number of stations. In this case, inspection at downstream operations can detect bad features made by upstream machines. We call this quality information feedback. A simple example of the quality information feedback in 2M1B systems is when M1 produces defective features but does not have inspection and M2 has inspection and it can detect bad features made by M1. In this situation, as we demonstrate below, the yield of a line is a

function of the size of buffer. This is because when buffer gets larger, more material can accumulate between an operation (M_1) and the inspection of that operation (M_2). All such material will be defective if a persistent quality failure takes place. In other words, if buffer is larger, there tends to be more material in the buffer and consequently more material is defective.

In addition it takes longer to have inspections after finishing operations. We can capture this phenomenon with the adjustment of a transition probability rate of M_1 from state -1 to state 0. Let us define f_{q1} as a transition rate of M_1 from state -1 to state 0 when

there is a quality information feedback and f_1 as the transition rate without the quality information feedback. The adjustment can be done in a way that the yield of M_1 is the same as $Z_{g1}Z_{b1} + Z_{b1}$

where Z_{b1} : the expected number of bad parts generated by M_1 while it stays in state -1. Z_{g1} : the expected number of good parts produced by M_1 from the moment when M_1 leaves the -1 state to the next time it arrives at state -1. Suppose that M_1 has been in state 1 for a very long time. Then all parts in the buffer B are non-defective. Suppose that M_1

goes to state -1. Defective parts will then begin to accumulate in the buffer. Until all the parts in the buffer are defective, the only way that M_1 can go to state 0 is due to its own inspection or its own operation failure. Therefore, the probability of a transition to 0 before M_1 finishes a part is eventually all the parts in the buffer are bad so that defective parts reach M_2 . Then, there is another way that M_1 can move to state 0 from state -1: quality information feedback. The probability that the inspection at M_2 detects a nonconformity made by M_1 is

$$\frac{f_1}{\mu_1} \equiv \chi_{11}$$

From (10), the yield of M_1 is

$$\frac{P(1)}{P(1) + P(-1)} = \frac{f_1^q}{f_1^q + g_1} \quad (64)$$

where $1/h_{21}$ is the mean time until the inspection at M_2 detects a bad part made by M_1 after M_2 receives the bad part.

The expected value of the number of bad parts produced by M_1 before it is stopped by either operational failures or quality information feedback is

$$Z_1^b = [\chi_{11} + 2\chi_{11}(1 - \chi_{11}) + 3\chi_{11}(1 - \chi_{11})^2 + \dots + w\chi_{11}(1 - \chi_{11})^{w-1}] \\ + [(w+1)(1 - \chi_{11})^w \chi_{21} + (w+2)(1 - \chi_{11})^{w+1} \chi_{21}(1 - \chi_{21}) + \dots] \quad (65)$$

where w is average inventory in the buffer B . This is an approximate formula since we simply use the average inventory rather than averaging the expected number of bad parts produced by M_1 depending on different inventory levels w_i . After some mathematical manipulation,

$$Z_1^b = \frac{1 - (1 - \chi_{11})^w}{\chi_{11}} - w(1 - \chi_{11})^w \\ + \frac{(1 - \chi_{11})^w \chi_{21} [(w+1) - w(1 - \chi_{11})(1 - \chi_{21})]}{[1 - (1 - \chi_{11})(1 - \chi_{21})]^2} \quad (66)$$

On the other hand, Z_1^g is given as

$$Z_1^g = \frac{\mu_1}{p_1 + g_1} + \frac{p_1}{p_1 + g_1} \frac{\mu_1}{p_1 + g_1} + \left(\frac{p_1}{p_1 + g_1}\right)^2 \left(\frac{\mu_1}{p_1 + g_1}\right) \dots = \frac{\mu_1}{g_1} \quad (67)$$

By setting $\frac{f_1^q}{f_1^q + g_1} = \frac{Z_1^g}{Z_1^g + Z_1^b}$ we have

$$f_1^q = \frac{\mu_1}{\frac{1 - (1 + w\chi_{11})(1 - \chi_{11})^w}{\chi_{11}} + \frac{(1 - \chi_{11})^w \chi_{21} [1 + w(\chi_{21} + \chi_{11} - \chi_{21}\chi_{11})]}{[1 - (1 - \chi_{11})(1 - \chi_{21})]^2}} \quad (68)$$

Since the average inventory is a function of f_1^q and f_1^q is dependent on the average inventory, an iterative method is required to determine these values.

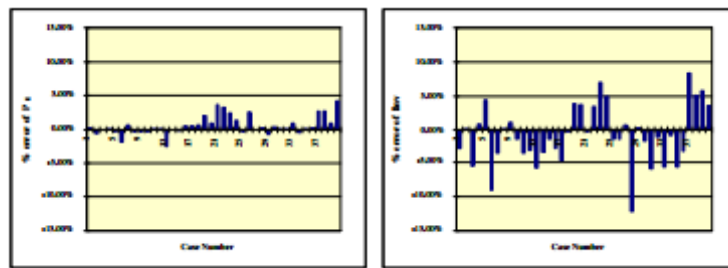


Fig. 6 Validation of the quality information feedback formula

Figure 6 shows the comparison of the effective production rate and the average inventory from the analytic model and the simulation. 50 cases are generated by selecting different machine and buffer parameters and % errors are plotted in the y-axis. The parameters for these cases are given in Appendix B. % errors in the effective production rate and average inventory are calculated using equations (62) and (63) respectively. The average absolute value of the % error in PE and x estimations is 1.01% and 3.67% Respectively.

INSIGHTS FROM NUMERICAL EXPERIMENTATION

In this section, we perform a set of numerical experiments to provide intuitive insight into the behaviour of production lines with inspection. The parameters of all the cases are presented in Appendix B.

BENEFICIAL BUFFER CASE SYSTEM

Production rates having quality information feedback means having more inspection than otherwise. Therefore, machines tend to stop more frequently. As a result, the total production rate of the line decreases. However, the effective production rate can increase since added inspections prevent the making of defective parts. This phenomenon is shown in Figure 7. Note

that the total production rate PT without quality information feedback is consistently higher than PT with quality information feedback regardless of buffer size and the opposite is true for the effective production rate PE. Also it should be noted that in this case, both the total production rate and the effective production rate increase with buffer size, with or without quality information feedback.

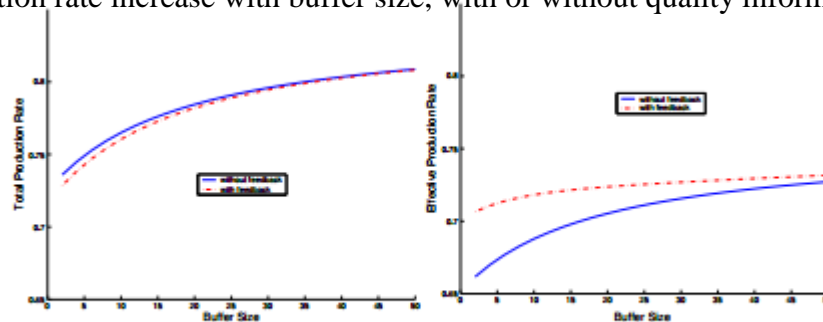


Fig. 7 Production Rates with/without Quality Information Feedback

System yield and buffer size Even though a larger buffer increases both total and effective production rates in this case, it decreases yield. As explained in Section 4, the system yield is a function of the buffer size if there is quality information feedback. Figure 8 shows system yield decreasing as buffer size increases when there is quality information feedback. This happens because when the buffer gets larger, more material accumulates between an operation and the inspection of that operation. All such material will be defective when the first machine is at state -1 but the inspection at the first machine does not find it. This is a case in which a smaller buffer improves quality, which is widely believed to be generally true. If there is no quality information feedback, then the system yield is independent of the buffer size (and is substantially less).

HARMFUL BUFFER CASE

Production rates typically, increasing the buffer size leads to higher effective production rate. This is the case in Figure 7. But under certain conditions, the effective production rate can actually decrease as buffer size increases. This can happen when The first machine produces bad parts frequently: this means g_1 is large. The inspection at the first machine is poor or non-existent and inspection at the second machine is reliable: this means h_1 h_2 or f_1 f_2 .? There is quality

information feedback. The isolated production rate of the first machine is higher than that of the second machine:

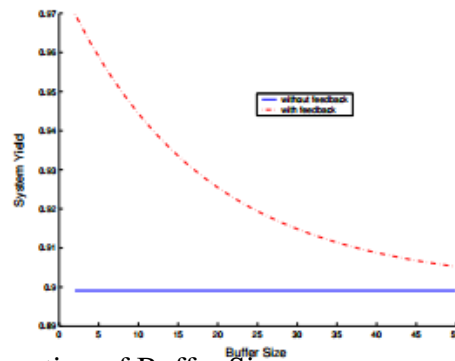


Fig. 8 System Yield as a Function of Buffer Size

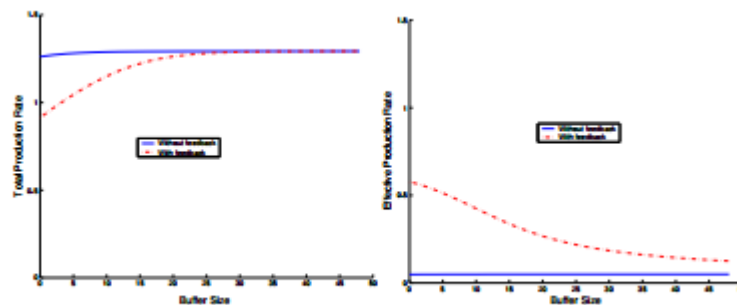


Fig. 9 Total Production Rate and Effective Production Rate

Figure 9 shows a case in which a buffer size increase leads to a lower effective production rate. Note that even in this case, total production rate monotonically increases as buffer size increases.

SYSTEM YIELD

The system yield for this case is shown in Figure 10.

Note that the yield decreases dramatically as the buffer size increases. In

this case, the decrease of the system yield is more than the increase of the total production rate so that the effective production rate monotonically decreases as buffer size gets bigger.

How to improve quality in a line with persistent quality failures There are two major ways to improve quality . One is to increase the yield of individual operations and the other is to perform more rigorous inspection. Having extensive preventive maintenance on manufacturing equipment and using robust engineering techniques to stabilize operations have been suggested as tools to increase yield of individual operations. Both approaches

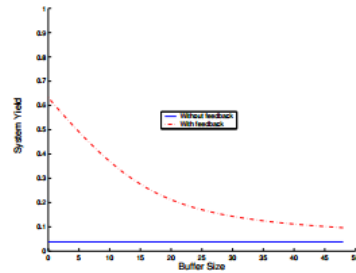


Fig. 10 System Yield as a Function of Buffer Size

increase the Mean Time to Quality Failure (MTQF) (i.e. decrease g). On the other hand, the inspection policy aims to detect bad parts as soon as possible and prevent their flow toward downstream operations. More rigorous inspection decreases the mean time to detect (MTTD) (i.e. increases h and therefore increases f). It is natural to believe that using only one kind of method to achieve a target quality level would not give the most cost efficient quality assurance policy. Figure 11 indicates that the impact of individual operation stabilization on the system yield decreases as the operation becomes more stable. It also shows that effect of improving inspection (MTTD) on the system yield decreases. Therefore, it is optimal to use a combination of both methods to improve quality.

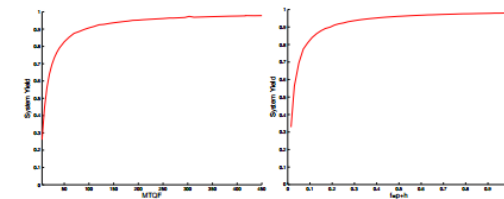


Fig. 11 Quality Improvement

HOW TO INCREASE PRODUCTIVITY & PRODUCTION OF A PRODUCT

Improving the stand-alone throughput of each operation and increasing the buffer space are typical ways to increase the production rate of manufacturing systems. If operations are apt to have quality failures, however, there may be other ways to increase the effective production rate: increasing the yield of each operation and conducting more extensive inspections. Stabilizing operations, thus improving the yield of individual operations, will increase effective throughput of a manufacturing system regardless of the type of quality failure. On the other hand, reducing the mean time to detect (MTTD) will increase the effective production rate only if the quality failure is persistent but it will decrease the effective production rate if the quality failure is Bernoulli. This is because the quality of each part is independent of the others when the quality failure is Bernoulli. Therefore, stopping the line does not reduce the number of bad parts in the future. In a situation in which machines produce defective parts frequently and inspection is poor, increasing inspection reliability is more effective than increasing buffer size to boost the effective production rate. Figure 12 shows this. Also, in other situations in which machines produce defective parts frequently and inspection is reliable, increasing machine stability is more effective than increasing buffer size to enhance effective production rate.

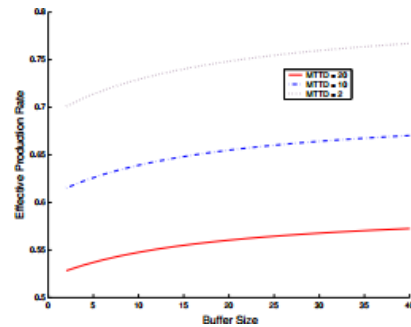


Fig. 12 Mean Time to Detect and Effective Production Rate

FUTURE SCOPES

The 2-Machine-1-Buffer (2M1B) model with $16 = 2$ is analyzed in [14]. This case is more challenging because the number of roots of the internal transition equations depends on parameters of machine. A more general 2M1B model with multiple-yield quality failures (a mixture of Bernoulli- and persistent-type quality failures) should also be studied. A long line analysis using decomposition is under the development. Refer to Kim [14] for more detailed information.

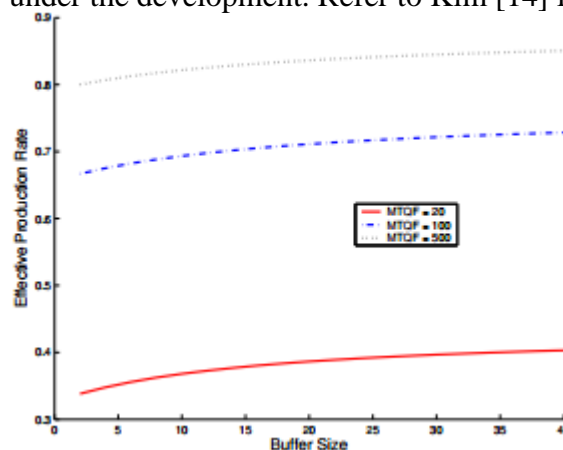


Fig. 13 Quality Failure Frequency and Effective Production Rate

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